Aji and Goldenfeld Reply: In a recent letter [1], we analyzed the critical dynamics of the superconducting to normal phase transition in zero magnetic field. We explained Monte Carlo (MC) simulation results [2] in both strong and weak screening limits, where the dynamic critical exponent was found to be $z_{MC} \sim 2.7$ and $z_{MC} \sim 1.5$ respectively. These results, taken at face value were surprising, departing strongly from scaling expectations based on model A dynamics that $z \sim 2$ [3]. We showed that the simulations do not measure the true dynamic exponent z and that in both the short and long range limits, provided the screening length is smaller than the system size, the dynamic exponent correctly inferred from the simulations is $z \sim 2$, thus removing the discrepancy.

In the preceding Comment [4], the author asserts that in our theory, the vorticity has the wrong scaling dimension, inconsistent with standard scaling theory and not supported by MC data. Second, he asserts, without justification, that the identification of MC time with real time is correct, and that our proposal that they are not the same is an influence of the discreteness of the lattice.

In fact, the non-equivalence of MC time and real time is a genuine effect, but arises due to an implementation of a dynamic MC algorithm that does not properly account for the scaling of J. Here, we show that our theory predicts a scale dependent coupling constant $J(L)$, and that the standard finite size scaling results are indeed satisfied, contrary to the statement in the Comment.

In terms of the vorticity, $\vec{n} = \vec{\nabla} \times \vec{\nabla}\theta$, the free energy is, $F = \beta \sum_{i,j} \vec{n_i}.\vec{n_j} G_{ij}[\lambda_0]$, where the lattice Green function is

$$G_{ij}[\lambda_0] = J \frac{(2\pi)^2}{L^3} \sum_{\vec{k}} \frac{\exp[i\vec{k}\cdot(\vec{r_i} - \vec{r_j})]}{2\sum_m^3 [1 - \cos(k_m)] + \lambda_0^{-2}} \quad (1)$$

Here J is the coupling constant, λ_0 is the screening length and $\beta = 1/k_B T$. In the long range case, $\lambda_0 \to \infty$, the $G_{ij} \sim J/|\vec{r_i} - \vec{r_j}|$, while in the short range case, $G_{ij} \sim J\delta(\vec{r_i} - \vec{r_j})$. The free energy density scales as L^{-d} ($F \sim (L/\xi)^d$), θ, the phase, has a trivial scaling dimension of 0 and $n \sim \xi^{-2}$. In the long range case, since $G_{ij} \sim J/\xi$, we get $J \sim \xi^2/L^3$ where ξ is the correlation length ($(L/\xi)^d \sim L^{2d}\xi^{-4}J/\xi$). In the short range case, where $G_{ij} \sim J/L^d$, a similar analysis yields $J \sim \xi$. In the dynamic MC simulations, the scaling of J is not accounted for and leads to scaling dimensions of n other than -2. If one ignored the scaling of J, the above analysis would yield $n \sim \xi^{-x}$ and $\nabla\theta \sim \xi^{1-x}$, where x is $5/2$ ($(L/\xi)^d \sim L^{2d}\xi^{-2x}/\xi$) in the weak screening limit and $x = 3/2$ in the strong screening limit. The superfluid density is obtained from, $F = \beta \int d\vec{r} \rho |\vec{\nabla}\theta|^2$. Given the scaling dimension of $\nabla\theta$ above, and assuming that J does not scale, $\rho \sim J\xi^{2x-2-d}$ ($(L/\xi)^d \sim \rho L^d \xi^{2-2x}$), which would disagree with standard scaling theory [5].

This discrepancy in the scaling dimensions arises only in MC simulations where a time dependent variable is measured, such as a two-time correlation function. The superfluid density in static MC simulations, for example, is obtained from the equal time current-current correlation function in the phase representation of the Villain model. Unlike the vorticity, the phase variable does not scale with system size ($\theta \sim L^0$). This computation of the correlation function reproduces the standard scaling form for ρ.

The non-equivalence of real time and MC time arises from using the same value of J for all lattice sizes L [6], and as explained in our letter, is a consequence of the fact that in a single MC time step, regardless of L, an equal change must be made to the voltage pulse produced by the vortex loops and not the vortex loops themselves. In dynamic MC simulations, if we scaled J appropriately, we would obtain $z \sim 2$, for strongly screened interactions. However, one usually does not know ahead of time what the scale dependence of $J(L)$ is.

In our theory, the scaling form for ρ is calculated from $\rho \sim J(L)L^{2x-d-2}$. In the case of short ranged interaction and the physically relevant case of weak screening ($\lambda_0 < L$), $x = 3/2$ and $J \sim L$ ($\xi \sim L$ at T_c), we get $\rho \sim L^{2-d}$ as expected. In the long range case, where λ_0 is set to infinity, the same argument yields $\rho \sim L^{2-d}$ since $x = 5/2$ and $J \sim L^{-1}$ in this case. For the magnetic permeability, including the scaling of J implies, $\mu \sim J^{-1}L^{d-2x}$. With this correction, we obtain the correct scaling form, $\mu \sim L^{d-4}$, in both the strong and weak screening limit.

In summary, our analysis is fully consistent with standard results, and explains the surprising results of the MC simulations.

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